Comparison of Experimental Data and Model Results for the Depressurization of an Air Tank

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Abstract

A laboratory experiment/demonstration and modeling study was developed for junior-level chemical engineering students that can serve as excellent training for the venting calculations performed in practice. A simple incompressible flow experiment was conducted in depressurizing an 11 gal (0.042 m^3) air tank from 80 to 22.5 psig (5.4 to 1.5 atm) through a sharp-edged orifice. A mathematical model was developed to predict the pressure of the air tank as a function of time using flow equations and an Euler integration. The model predicted the experimental data very well, with a maximum variance from the data of 14% for the initial time step of 0 - 75 s.

Keywords

Tank depressurization, overpressurization, vacuum failure, incompressible flow, modeling

Introduction

Students are likely to encounter a vast array of teaching techniques and styles during their academic careers, and this variety generally adds to the educational experience. However, if students were given a choice, they would most likely select an interactive teaching style for their classes. In a survey of 47 University of Michigan undergraduate engineering students (30 males, 17 females), Pomales-Garcia and Liu (2007) found that the students most preferred teaching that included examples, demonstrations, stories, websites, visual displays, group work, competitions and oral presentations. As engineering class sizes increase, the temptation for instructors is to move toward lecture-based learning (how can I possibly interact with all of these students?) (Hora *et al.* 2012). Notable exceptions to this practice exist, including the flipped classroom.

Student engagement through examples and classroom demonstrations is prominent among the student preferences. Johri and Olds (2011) note that some of the most essential skills in engineering come about through the use of tools and materials, and through interactions with other people. Fluid mechanics has been a popular subject for this type of classroom engagement, both in the laboratory and through classroom demonstrations. Kresta (1998) began using short demonstrations in the fluid mechanics classroom, and saw an increase in attendance from 30% to over 80%. Stern *et al.* (2006) developed a hands-on CFE educational interface for graduate engineering courses and laboratories. Loinger and Hermanson (2002) used an integrated experimental-analytical-numerical approach in the teaching of fluid mechanics, and student surveys showed that 90% of their students preferred this re-designed class to the traditional lecture class, while also obtaining a better understanding of the engineering fundamentals.

Storage tank overpressurization and vacuum failure are significant engineering problems. Overpressurization can cause tank rupture, leading to loss of containment, which may increase the risk of fire or even loss of life. Similarly, lowered tank pressures during tank evacuation could create a vacuum inside the tank, possibly causing the tank to implode (Tran 2016). Youtube shows a number of entertaining videos (for the viewer, and not the tank operator, engineer or owner) which demonstrate the potential consequences and dangers of overpressurization or vacuum failure (LiveLeak 2008, Wabash National 2014). Figure 1 shows an example of tank buckling due to accidentally induced internal vacuum. PetroWiki (2015) reviews vent system design and auxiliaries for storage tanks as an aid in helping to prevent these catastrophic events.

Figure 1. Tank Collapse Due to Accidentally Induced Internal Vacuum (de Paor 2010)

A laboratory experiment/demonstration and modeling study was developed for junior-level chemical engineering students that can serve as excellent training for the venting calculations performed in practice. In addition, the subsequent modeling study requires the students to utilize compressible flow equations and perform computer modeling, including the numerical integration of differential equations. The objective of this paper is to describe a simple experiment for the depressurization of a 11 gal (0.042 m^3) air tank through a 0.052 in (1.32 mm) sharp-edged orifice. Experimental data from the experiment were compared to computer model predictions. The computer model used an Euler integration to solve the differential equation.

Experimental

Apparatus

A photograph of the experimental apparatus is shown in Figure 2. The apparatus consisted of an 11 gal (0.042 m^3) Campbell-Hausfeld carbon steel air tank with a maximum pressure rating of 125 psig (8.5 atm above atmospheric) and a calculated actual volume of 11.6 gal (0.044 m^3). The tank was equipped with a $\frac{1}{4}$ in (6.4 mm) valve to initiate depressurization, a 0-160 psig (0-10.9 atm above atmospheric) pressure gauge and a ¼ in (6.4 mm) brass pipe plug. To form the sharp-edge orifice, a ¼ in (6.4 mm) brass pipe plug was drilled from both sides; a $\frac{5}{16}$ in (7.94

mm) square bottom drill was used to form the inlet cavity; a ¼ in (6.4 mm) partially square bottom drill was used to form the outlet cavity. The drilled holes formed a 0.01 in (0.25 mm) plate about midway through the pipe plug. This plate was drilled in its center with a 0.052 in (1.32 mm) diameter drill to complete the orifice. Figure 3 shows a photograph of the brass fitting.

Figure 2. Photograph of the Air Tank and Respective Attachments for Venting Data Collection

Figure 3. Photograph of the Brass Plug and Orifice, Shown from the Outlet Side

Experimental Procedure

Prior to experimentation, the tank was first checked for any defects. The tank was then pressurized to 65 psig (5.4 atm absolute) with shop air before moving the apparatus to the classroom. To begin the experiment, the ball valve was opened fully, and pressure was then measured as a function of time. To simplify the experiment, the time was initially recorded for each 5 psi (0.34 atm) reduction in pressure and later for each 2.5 psi (0.17 atm) reduction in pressure. The experiment ended when the tank reached a pressure of 7.5 psig (1.5 atm absolute).

Safety Concerns

Proper safety equipment for this experiment includes the wearing of safety goggles, long pants and protective gloves. Prior to tank venting, the orifice must be free of obstruction, and the path of the pressurized air must be clear to avoid damage to students and the surroundings.

Experimental Data

Table 1 presents the experimental data, collected as the time required to reach 5 psi, and later, 2.5 psi pressure changes in the tank.

Table 1. Experimental Data - Tank Pressure versus Time

Time (s)	Tank Pressure (psig)
0	65.0
10.32	60.0
18.94	55.0
30.45	50.0
48.39	45.0
66.29	40.0
93.96	35.0
114.44	30.0
130.65	27.5
144.75	25.0
158.52	22.5
170.39	20.0
181.26	17.5
197.84	15.0
223.46	12.5
245.45	10.0
263.47	7.5

Model Development

In performing a mass balance on the tank

$$m_i - m_o + m_g = m_a \tag{1}$$

Since the tank has no inlet streams or generation of air, Equation (1) reduces to

$$-m_o = m_a \tag{2}$$

The mass accumulated within the tank, m_a, may be expressed as a differential change in the mass of air in the tank using the continuity equation

$$m_a = \frac{dM}{dt} = \frac{d(\rho_t V)}{dt} \tag{3}$$

The density of the gas can be expressed as follows using the ideal gas equation

$$\rho_t = \frac{P_t(MW)}{RT} \tag{4}$$

Expanding Equation (3) to also include the ideal gas equation yields

$$m_a = \frac{d\left(\frac{P_t(MW)V}{RT}\right)}{dt} = -m_o \tag{5}$$

Finally, the differential pressure change as a function of changing mass flow rate is found by a rearrangement of Equation (5)

$$\frac{dP_t}{dt} = -\frac{m_o RT}{V(MW)} \tag{6}$$

The mass velocity of air leaving the tank may be calculated from the equation (McCabe *et al.* 2005)

$$G = \sqrt{\frac{2\gamma\rho_t P_t}{\gamma-1}\left(\frac{P_{vc}}{P_t}\right)^{1/\gamma}}\sqrt{1-\left(\frac{P_{vc}}{P_t}\right)^{[1-(1/\gamma)]}} \tag{7}$$

The constant γ is the specific heat ratio, which is 1.4 for air.

In Equation (7), the gas density within the tank, ρ_t, and pressure at the vena contracta, P_{vc}, were found using Equations (8) and (9), respectively

$$\rho_t = \frac{P_t(MW)}{RT} \tag{8}$$

$$P_{vc} = P_t(r_c) \tag{9}$$

In these equations, P_t is the absolute tank pressure, MW is the molecular weight of air, R is the ideal gas constant and T is the tank temperature. The parameter, r_c, is the critical pressure ratio, calculated by the equation (McCabe *et al.* 2005)

$$r_c = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{1-\frac{1}{\gamma}}} \tag{10}$$

For air r_c = 0.53. For $\frac{P_a}{P_t}$ < 0.53, where sonic velocity occurs, P_{vc} in Eq. 7 was set equal to the orifice pressure = 0.53 P_t; for $\frac{P_a}{P_t}$ > 0.53, P_{vc} was set equal to atmospheric pressure.
The mass flow from the tank was calculated using the mass velocity (G), the cross-sectional area of the orifice ($Ao = \frac{\pi d^2}{4}$), a dimensionless expansion factor (Y), and the orifice coefficient (C_d)

$$m_o = C_d Y G A o \tag{11}$$

The orifice coefficient was found using a linear regression of C_d versus $1 - P_r$ data from Linfield (2014). P_r is the pressure ratio of atmospheric pressure to absolute tank pressure, $\frac{P_{atm}}{P_t}$. The regression results in a fourth-order equation

$$C_d = 0.6219 + 0.0686(1 - P_r) + 0.7955(1 - P_r)^2 - 0.9285(1 - P_r)^3 + 0.2914(1 - P_r)^4 \tag{12}$$

According to the McCabe *et al*. (2005), if the critical pressure ratio, r_c, is less than 0.53 for air, the gas flow is sonic and $Y = 1$. For other r_c values, the expansion factor was computed as follows

$$Y = 1 - \frac{0.41 + 0.35\beta^4}{\gamma}\left(1 - \frac{P_{atm}}{P_t}\right) \tag{13}$$

To determine P as a function of time, the mass flow from the tank is first found using Equations (7) - (13). An Euler integration was then used to solve the differential equation in Equation (5).

Results and Discussion

Figure 4 presents a plot of the experimentally measured tank pressures and the model predicted pressures with time. There was minimal deviation in the mathematical model from the experimental data, indicating that the model is adequate. The mathematical model predicted slightly higher pressures (by an average of 14%) than the experimental data from 0 – 75 s, while the predicted values were only slightly lower than the experimental values for the time range of 100 – 175 s. The model matched the experimental data very well for times greater than 175 s. Due to the simplicity of the experiment, the system required minimal user interaction. Thus, errors were minimal.

Educational Use and Value

1. The experiment is simple and inexpensive. An 11 gal (0.042 m^3) air tank costs about \$40 and a digital pressure gauge costs about \$70; thus, for less than \$200, one can build an experimental apparatus which gives data which can be predicted by a mathematical model.
2. Critical and sub-critical nozzle flow occurs in the experiment. Students gain experience handling both.
3. The experiment can be conducted anywhere, including a classroom.

4. The modeling involves the solution of a 1st order differential equation using numerical methods. The computer program involves a logic statement which must be included to handle the transition from critical to non-critical flow as the tank pressure decreases.
5. The mathematical model fits experimental data very well, which eliminates the frustrating experience for students of explaining why the model does not predict the experimental data.

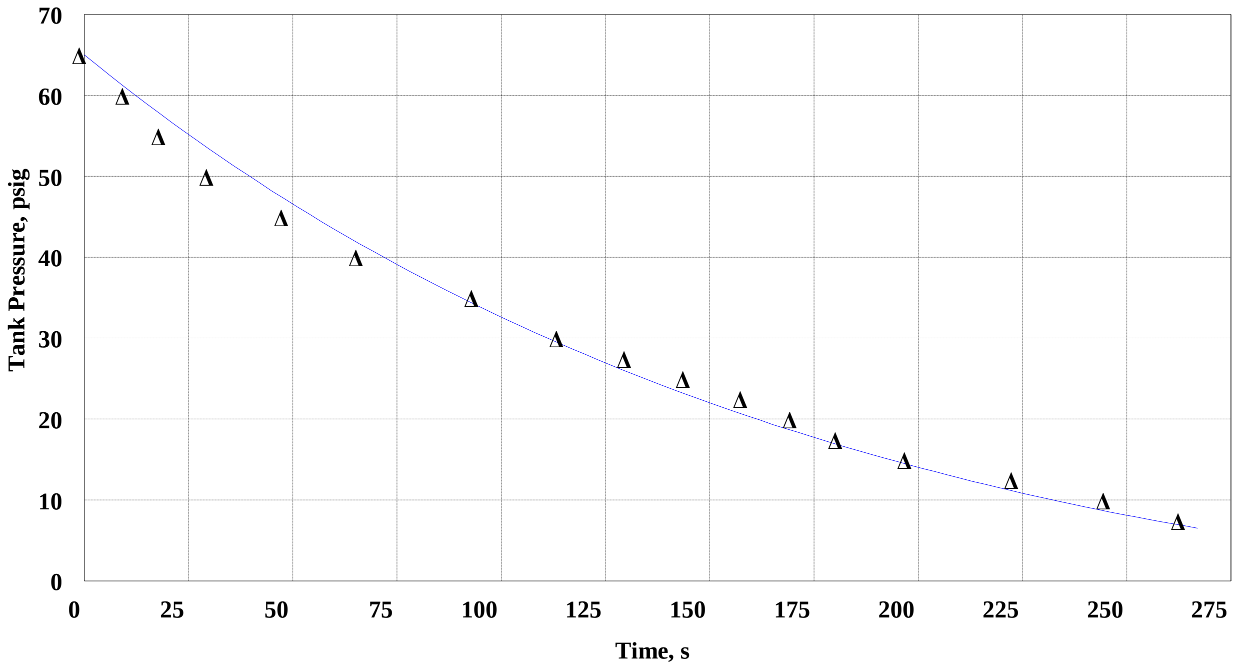

Figure 4. Experimental and Model Results from the Tank Depressurization

Conclusions

1. The mathematical model predictions and results fit the experimental data well throughout the depressurization time. The maximum variance of the model from the data was 14% for the initial time step of 0 - 75 s.
2. Errors in the experiment were most likely due to human error as a result of the lag between reading the pressure and corresponding time. This error has been minimized in an upgrade of the experimental system by the addition of a Measurement Computing data acquisition device, USB-TC-AI, driven by a 2 amp/12 volt source and connected to the USB port of a Dell Latitude E 5510 laptop computer. Omega TracerDAQPro software can be used to display and analyze the data.

Nomenclature (SI units shown)

Latin Symbols

A_o Cross sectional area of the orifice, m^2

C_d Orifice discharge coefficient, dimensionless
d Orifice diameter, m
$\frac{dM}{dt}$ Change of mass in the tank with respect to time, $\frac{kg}{s}$
$\frac{dP_t}{dt}$ Change in tank pressure with respect to time, $\frac{Pa}{s}$
G Mass velocity, $\frac{kg}{m^2 s}$
m_a Mass accumulation within the tank, $\frac{kg}{s}$
m_g Mass generation within the tank, $\frac{kg}{s}$
m_i Mass entering the tank, $\frac{kg}{s}$
m_o Mass exiting the tank, $\frac{kg}{s}$
MW Molecular weight of air, $\frac{kg}{kgMol}$
P_{atm} Atmospheric pressure, Pa
P_r Atmospheric to tank pressure ratio
P_t Tank pressure, Pa
P_{vc} Pressure at the vena contracta, Pa
R Gas constant, $\frac{cm^3 kPa}{gmol\ K}$
r_C Critical pressure ratio
T Tank temperature, K
V Tank volume, m^3
Y Gas expansion factor

Greek Symbols

β Diameter ratio
γ Specific heat ratio
ρ_t Air density, $\frac{kg}{m^3}$

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